

# Discussion

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# Arbitrary Binary Relations

Let  $R$  be an arbitrary binary relation,  $R \subseteq U \times U$ .

Let  $x$  be a member of  $U$ .

The  $R$ -*successor* set of  $x$ , denoted by  $R_s(x)$ , is defined as follows

$$R_s(x) = \{y \mid xRy\}.$$

Let  $X$  be a subset of  $U$ .

# Singleton Parameterized Approximations

The **singleton parameterized approximation** of  $X$  with the threshold  $\alpha$ ,  $0 < \alpha \leq 1$ , denoted by  $\text{appr}_\alpha^{\text{singleton}}(X)$ , is defined as follows

$$\{x \mid x \in U, \Pr(X|R_s(x)) \geq \alpha\},$$

where  $\Pr(X|R_s(x)) = \frac{|X \cap R_s(x)|}{|R_s(x)|}$  is the conditional probability of  $X$  given  $R_s(x)$ .

# Subset Parameterized Approximations

A **subset parameterized approximation** of the set  $X$  with the threshold  $\alpha$ ,  $0 < \alpha \leq 1$ , denoted by  $appr_{\alpha}^{subset}(X)$ , is defined as follows

$$\cup\{R_s(x) \mid x \in U, Pr(X|R_s(x)) \geq \alpha\}.$$

# Concept Parameterized Approximations

A **concept parameterized approximation** of the set  $X$  with the threshold  $\alpha$ ,  $0 < \alpha \leq 1$ , denoted by  $appr_{\alpha}^{concept}(X)$ , is defined as follows

$$\cup\{R_s(x) \mid x \in X, Pr(X|R_s(x)) \geq \alpha\}.$$

# Incomplete Data Sets

Two types of missing attribute values:

*lost*, denoted by "?", (e.g., the value was erased)

and

*"do not care" conditions*, denoted by "\*", (such a value may be any value of the attribute).

# An Incomplete Data Set

Case	Attributes			Decision
	Temperature	Headache	Cough	Flu
1	normal	no	*	no
2	?	no	no	no
3	normal	*	no	yes
4	normal	no	?	no
5	high	yes	*	yes
6	high	yes	yes	no
7	high	?	yes	yes
8	high	yes	yes	yes

# Characteristic Relation

$$K(1) = K(4) = \{1, 3, 4\},$$

$$K(2) = \{1, 2, 3\},$$

$$K(3) = \{1, 3\},$$

$$K(5) = K(6) = K(8) = \{5, 6, 8\},$$

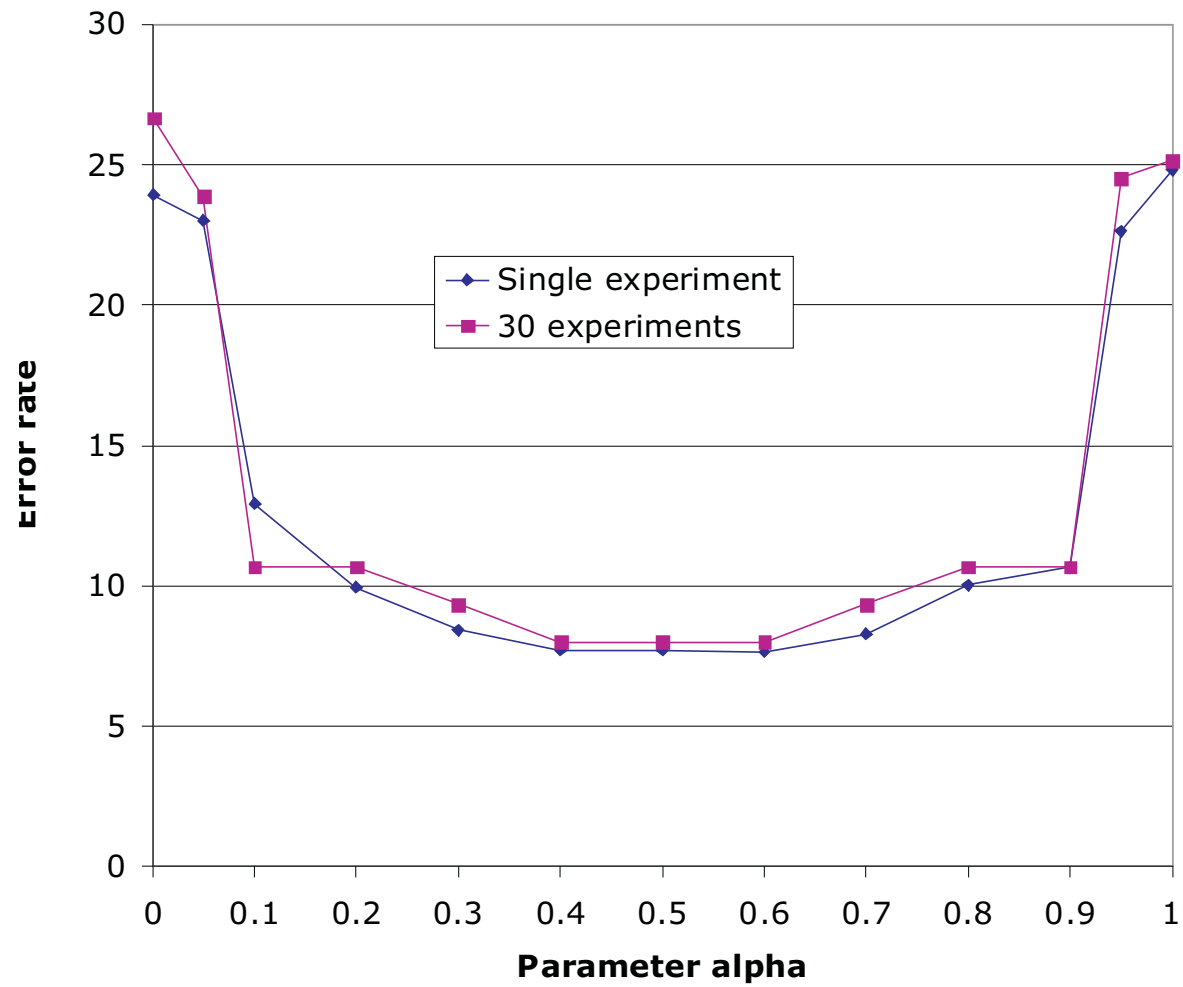
$$K(7) = \{5, 6, 7, 8\}.$$

$$R = \{(1, 1), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1), (4, 3), (4, 4), (5, 5), (5, 6), (5, 8), (6, 5), (6, 6), (6, 8), (7, 5), (7, 6), (7, 7), (7, 8), (8, 5), (8, 6), (8, 8)\}$$

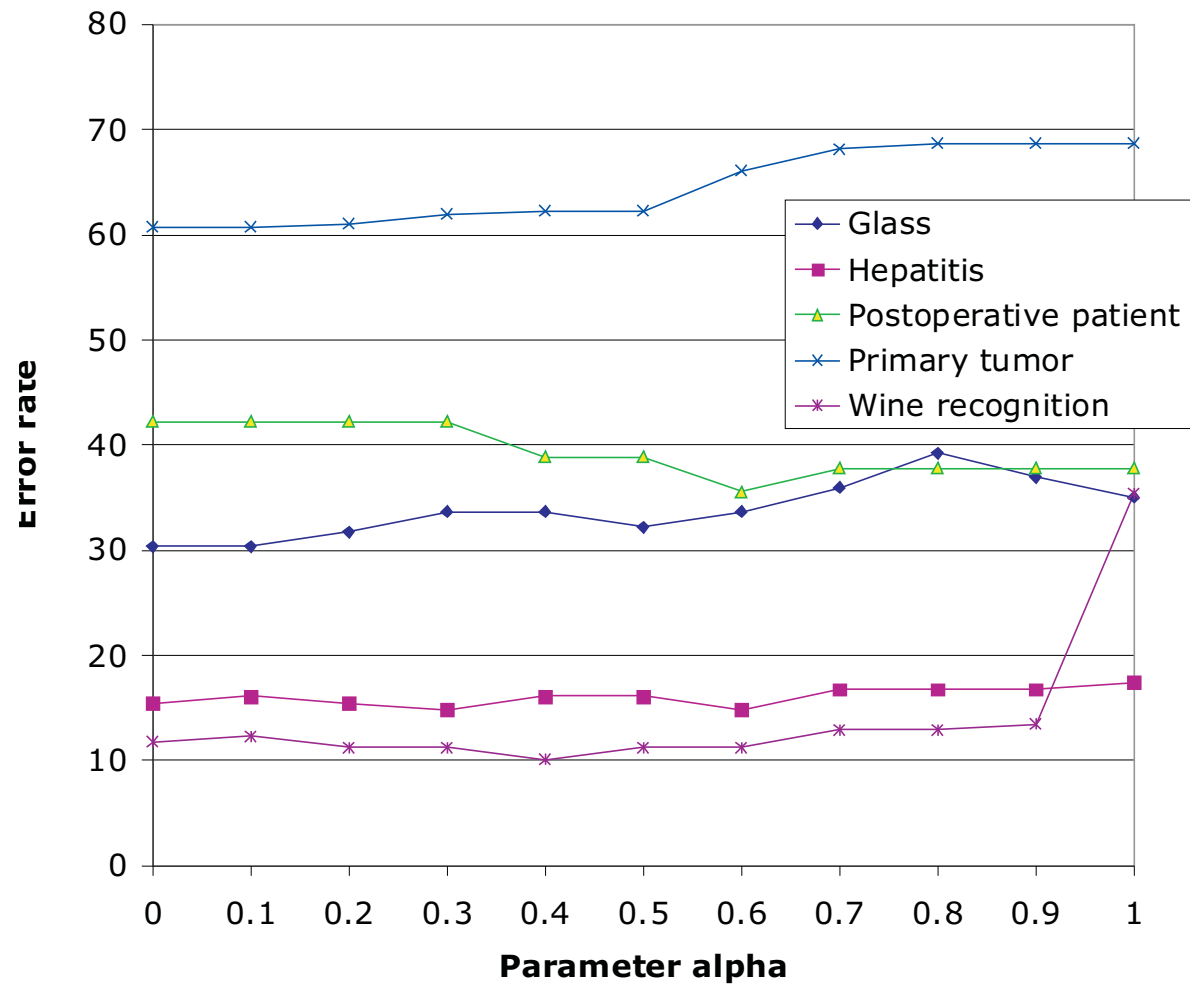
$R$  is not symmetric (since  $(5, 7)$ ) and not transitive (since  $(2, 1)$  and  $(1, 4)$ ).



# Iris Data Set



# Five Other Data Sets



# Iris Data Set (56% consistent)

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Concept	Number of distinct parameterized approximations
Class, Iris-setosa	2
Class, Iris-versicolor	6
Class, Iris-viginica	5

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